ample, the best value of W would probably be less than one and possibly even zero.

This "near-optimal" procedure is based on the optimal composite feedforward-feedback control study of Luecke and McGuire (*2*). The same method could be applied to Figure 6 if the inlet composition were actually not available for measurement. If the total control effort available is limited by manipulative input constraints, the constraints can be included directly (as in saturation clipping) or treated in terms of a penalty function.

In this note, a straightforward signal flow graph analysis has provided valuable insight into an involved multiloop control scheme. It is an example of the value of graphical signal flow diagrams as analytical aids in control system design.

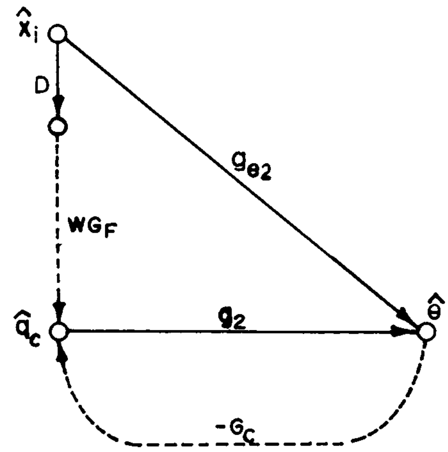

Fig. 7. Feedback temperature-feedforward disturbance control.

ACKNOWLEDGMENT

This analysis was partially supported by the National Science Foundation Grant GK-3735.

NOTATION

$\wedge$ = Laplace transform of the variable perturbation from steady state
D = operator representing the time delays L_1 and L_2, the sampler T_1, and zero-order hold h_1 of reference (*1*)
f_{10} = a variable generated by the controlled process system
F_1 = a controller function of the predictive-feedback scheme
F_2 = a controller function of the predictive-feedback scheme
g_c = a controller function of the predictive-feedback scheme
g_1, g_2, g_3, g_{e2} = process transfer functions given in reference (*1*)
G_C = a feedback controller function
G_F = a feedforward controller function
q_c = coolant flow rate
w = weighting constant of composite feedforward-feedback control
x = reactor composition variable
x^* = delayed, sampled reactor composition variable
x_i = inlet composition variable
θ = reactor temperature

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Comments on "A Phenomenological Interpretation and Correlation of Drag Reduction"

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In the above-mentioned paper (*1*), Astarita, Greco, and Nicodemo proposed the following correlation for turbulent drag reduction in dilute polymer solutions:

$$\frac{f}{f_0} = \beta\,(N_{De}) \qquad (1)$$

Here f is the measured friction factor, f_0 the friction factor predicted from the usual Newtonian relationship (*2*), and $N_{De} = \theta\, U/D\, N_{Re}^{0.75}$ is the Deborah number (*1*). In the development of Equation (1), it was assumed that θ, the characteristic fluid time, is constant for a given polymer solution.

According to the aforementioned authors, $\beta(N_{De})$ should be a universal function; that is, independent of solution characteristics, Reynolds number, and tube diameter. This being the case, Equation (1) may be rewritten as

$$\frac{f}{f_0} = \beta\left(\frac{N_{De}}{N_{De,0.5}}\, N_{De,0.5}\right)$$

where $N_{De,0.5}$ is the unique Deborah number corresponding to $\beta = 0.5$. It follows that

$$\frac{f}{f_0} = \beta\left[\frac{\dfrac{U}{D} N_{Re}^{0.75}}{\left(\dfrac{U}{D} N_{Re}^{0.75}\right)_{,0.5}} N_{De,0.5}\right] = \beta'\left(\frac{\omega}{\omega_{,0.5}}\right) \qquad (2)$$

where $\omega = U/D\, N_{Re}^{0.75}$ is a characteristic frequency.

Astarita and his co-workers verified Equation (2) by plotting f/f_0 versus $\omega/\omega_{,0.5}$ for five concentrations of ET-597 in three different-diameter tubes, obtaining a single curve. However, as the authors noted, some conceptual difficulties arise with regard to the definition of N_{Re} for the more concentrated solutions where pseudoplastic effects are present. Furthermore, as Equation (2) is now written, a value of β less than unity does not necessarily correspond to true drag reduction. Finally, we have the difficulty in evaluating $\omega_{,0.5}$ (corresponding to $\beta = 0.5$) for slightly drag-reducing systems.

These problems may be eliminated in the case of dilute

Newtonian solutions, characterized by

$$\eta_{\text{Rel.}} \leq 1.2 \qquad (3)$$

For the ET-597 solutions of Astarita, Greco, and Nicodemo's study, Equation (3) limits the maximum allowable concentration to 30 ppm.* For Separan AP-30 (Dow), the allowable concentration is 125 ppm. (4); for Polyox WSR-301 (Union Carbide) 100 ppm, and for Polyox N-750 600 ppm. (5, 6). Under the restriction of Equation (3), β may be written as f/f_s, where f_s is the friction factor based on the solvent Reynolds number, with a maximum error of less than 5%.† Equation (1) is then

Equation (1) is then

$$\frac{f}{f_s} = \beta\,(N_{De}) = \beta\left(\frac{\theta\eta_s^{0.75}}{\eta^{0.75}}\,\frac{U}{D}\,N_{Re_s}{}^{0.75}\right) \qquad (4)$$

where N_{Re_s} is the solvent Reynolds number. Using an argument identical with that leading to Equation (2), we find

$$\frac{f}{f_s} = \beta''\left(\frac{\omega_s}{\omega_{s,\beta}}\right) \qquad (5)$$

where $\omega_s = U/D\ N_{Re_s}{}^{0.75}$, and $\omega_{s,\beta}$ is the value of ω_s at an arbitrary value of f/f_s. The point here is that, according to the theory, an infinite number of reduced universal curves exist, corresponding to different values of $\omega_{s,\beta}$ [or $\omega_{,\beta}$ in Equation (2)]. It would seem advantageous, in general, to utilize a reduced curve with a more easily attainable value of β, say 0.80. In this case, we have

$$\frac{f}{f_s} = \beta''\left(\frac{\omega_s}{\omega_{s,0.8}}\right) \qquad (6)$$

Equation (6) offers the following advantages in use as compared to Equation (2):

(1) A value of β'' less than unity corresponds to an actual reduction in drag with respect to the solvent.

(2) An accurate value of the solution viscosity is not required; we need only know Equation (3) is satisfied.

(3) $\omega_{s,0.8}$ would be, in general, easier to obtain than $\omega_{,0.5}$.

These comments, of course, presuppose the basic aspects of the theory to be correct. Further experiments are needed on this point. The essential diameter dependence of drag reduction predicted by Equations (2) and (6) has been well verified (7).

NOTATION

D = tube diameter
f = measured friction factor
f_0 = friction factor predicted from Newtonian correlations, using solution viscosity to obtain N_{Re}
f_s = friction factor predicted from Newtonian correlations, using solvent viscosity to obtain N_{Re}
N_{De} = Deborah number, defined following Equation (1)
$N_{De,0.5}$ = value of Deborah number corresponding to $f/f_0 = 0.5$
N_{Re} = Reynolds number, based on solution viscosity
N_{Re_s} = Reynolds number, based on solvent viscosity
U = average velocity
β = function defined in Equation (1)
β' = function defined in Equation (2)
β'' = function defined in Equation (5)
η = solution viscosity
η_s = solvent viscosity
$\eta_{\text{Rel.}}$ = η/η_s
θ = characteristic time of solution
ω = frequency, defined following Equation (2)
$\omega_{,0.5}$ = value of ω at $\beta' = 0.5$
ω_s = frequency, defined following Equation (5)
$\omega_{s,\beta}$ = value of ω_s at an arbitrary value of β

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* This figure is based on Oliver's value of $\eta_{\text{Rel.}} = 1.70$ for a freshly prepared 0.01% solution (3). For a slightly degraded solution, the allowable concentration would increase significantly.

† With $\eta_{\text{Rel.}} \leq 1.4$, this increases to about 9%.

Similarity Transformations for Continuous Cylindrical Surfaces in Axial Motion

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The article by Vasudevan and Middleman (2) makes a valuable contribution by presenting the velocity, temperature, and concentration profiles for a fluid surrounding a continuous circular cylinder in axial motion. It appears, though, that the assumption

$$\pi_i = \pi_i(\eta) \qquad (1)$$

which was used in deriving the equation

$$\frac{d^2\pi_i}{d\eta^2} + \frac{\Lambda_i}{\eta} f \frac{d\pi_i}{d\eta} = 0 \qquad (2)$$

is not justified.

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